

## **MATH-807 Commutative Algebra**

**Credit Hours: 3-0**

**Prerequisite:** Students must know the basic concepts of rings, quotient rings, integral domains and fields. Those students who have done Algebra / or equivalent will be preferred.

**Course Objectives:** This course aims to introduce students to the concepts of modules over commutative rings, Noetherian rings and modules, Artinian rings and valuation rings.

**Detailed Course Contents:** Rings, ideals, operations on ideals, radical of an ideal, nilradical, Jacobson radical, ideal quotient, local rings, prime avoidance lemma, modules, module over commutative rings, submodules, operations on submodules, finitely generated modules, free module, annihilator of an element of a module, cyclic modules, quotient modules, direct sum and product of modules, module homomorphisms, short exact sequences, tensor product of modules, rings and modules of fractions, extended and contracted ideals in rings of fractions, Integral dependence, the going-up theorem, valuation rings, chain conditions, Noetherian rings and modules, Nakayama's lemma, primary decomposition, primary decomposition in Noetherian rings.

**Learning Outcomes:** On successful completion of this course, students will know Rings, ideals, operations on ideals, radical of an ideal, nilradical, Jacobson radical, ideal quotient, local rings, modules, submodules, operations on submodules, finitely generated modules, freemodules, direct sum and product of modules, module homomorphisms, isomorphism theorems of modules, tensor product of modules, rings and modules of fractions, Integral dependence, valuation rings, primary decomposition Noetherian rings and modules.

### **Recommended Books**

- 1) M. F. Atiyah, and I. G. Macdonald, Introduction to Commutative Algebra, Addison- Wesley, 1994. ISBN: 9780201407518.
- 2) D. Eisenbud, "Commutative Algebra with a View Toward Algebraic

Geometry", Springer, New York, 1995.

- 3) Thomas W. Hungerford, Algebra, Springer-Verlag, New York Inc. 1974.
- 4) David S Dummit, Richard M. Foote, Abstract Algebra, (3rd Ed.), 2004, John Wiley & Sons.

### ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown	
Week	Topics
1	Commutative rings, integral domains, Euclidean domains, the greatest common divisor of two elements of a ring, and related theorems.
2	PID's, UFD's, and related theorems, properties of the polynomial rings, polynomial rings over fields.
3	Existence of maximal ideals of a commutative ring with unity, local rings.
4	Nilradical, Jacobson radical, related theorems, operations on ideals.
5	Radical of an ideal, ideal quotient, comaximal ideals, the Chinese Remainder Theorem.
6	Monomial ideals, operations on monomial ideals, radical of a monomial ideal, colon ideal of two monomial ideals.
7	Module over commutative rings, examples, submodules, operations on submodules.
8	Finitely generated modules, cyclic modules, Nakayama's lemma, free modules,

	torsion modules, torsion free modules.
9	<b>Mid Semester Exam</b>
10	Quotient modules, module homomorphisms, isomorphism theorems of modules. Direct sum and direct product of modules,
11	short exact sequences, tensor product of modules.
12	Rings and modules of fractions, localization.
13	Primary decomposition.
14	Chain conditions, Noetherian rings, and modules
15	Artinian rings and modules.
16	Primary decomposition in Noetherian rings.
17	Review
18	<b>End Semester Exam</b>